

Title: Adding or Subtracting Rational Expressions
Class: Math 100 or Math 107 or Math 111
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Instructions to Tutor: Read instructions and follow all steps for each problem exactly as given.
Keywords/Tags: adding rational expressions, subtracting rational expressions, rational expressions

Adding or Subtracting Rational Expressions

Purpose: This is intended to refresh your skills in adding and subtracting rational expressions.

Activity: Work through the following activity and examples. Do all of the practice problems before consulting with a tutor.

- Adding or subtracting rational expressions is done in the same way as adding or subtracting fractions. They must have a common denominator.
- Once you have rewritten your fractions so they all have the same denominator, then rewrite as a single fraction by adding and subtracting the numerators all over the common denominator.
- Once the numerator has been simplified by combining like terms, then, if possible, factor the numerator (your denominator should be left in factored form) and reduce.
- The rules for simplifying rational expressions are the same as the rules for simplifying fractions: **Only common factors may be reduced.**
- It is harder to tell when a rational expression has been **factored**:

Consider $2x^2 - x - 15$ The last operations are addition/subtraction. $2x^2$, $-x$, and -15 are terms (expressions which are added or subtracted), not factors.	=	$(2x+5)(x-3)$ The last operation is the multiplication between the $)$. $(2x+5)$ and $(x-3)$ are factors (expressions which are multiplied).
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Consider $8x^3 - 4x^2 - 60x$ $8x^3$, $-4x^2$, and $-60x$ are terms . These can not be reduced.	=	$4x(2x+5)(x-3)$ 4 , x , $(2x+5)$, and $(x-3)$ are factors . These can be reduced.
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Example 1

$$\frac{5c}{c+6} + \frac{30}{c+6}$$

$$\frac{5c+30}{c+6}$$

$$\frac{5(c+6)}{c+6}$$

Example 2

$$\frac{10}{3x-7} - \frac{5}{3x-7}$$

$$\frac{10-5}{3x-7}$$

$$\frac{5}{3x-7}$$

Practice 1 $\frac{4y}{y+3} + \frac{12}{y+3}$

Practice 2 $\frac{y}{y+3} - \frac{-3}{y+3}$

Did you get 4?

Did you get 1?

- When terms are being added, we can rewrite their order using the commutative property of addition: $3 + x = x + 3$
- But, subtraction is not commutative: $3 - x \neq x - 3$
- However, we can factor out a -1 : $3 - x = -1(-3 + x) = -1(x - 3)$
So, when you need to rewrite the order of two terms being subtracted, factor out a -1 .
- Use the following properties to rewrite where you put your negative signs (by convention, we try not to leave any in denominators).

$$\frac{A}{-B} = \frac{-A}{B} = -\frac{A}{B} \quad \text{and} \quad -\frac{A}{-B} = -\frac{-A}{B} = \frac{-A}{-B} = \frac{A}{B}$$

Example 3

$$\frac{10}{3x-7} - \frac{5}{7-3x}$$

$$\frac{10}{3x-7} - \frac{5}{-1(3x-7)}$$

$$\frac{10}{3x-7} + \frac{5}{3x-7}$$

$$\frac{15}{3x-7}$$

Practice 3 $\frac{y}{y-3} + \frac{3}{3-y}$

Did you get -1 ?

Example 4a $\frac{5}{a+1} + \frac{4}{3a+3}$

$$\frac{5}{(a+1)} + \frac{4}{3(a+1)}$$

$$\frac{5}{(a+1)} \cdot \frac{3}{3} + \frac{4}{3(a+1)}$$

$$\frac{15+4}{3(a+1)}$$

$$\frac{19}{3(a+1)}$$

Factor the denominators. The LCD is $3(a+1)$

Multiply by 1, in the form of the "missing" factor over itself: $\frac{3}{3}$

Example 4b $\frac{3a}{2a+6} - \frac{a-3}{a+3}$

$$\frac{3a}{2(a+3)} - \frac{(a-3)}{a+3} \cdot \frac{2}{2}$$

$$\frac{3a-2(a-3)}{2(a+3)}$$

$$\frac{3a-2a+6}{2(a+3)}$$

$$\frac{a+6}{2(a+3)}$$

Practice 4 $\frac{z+7}{3z-9} - \frac{z-6}{z-3}$

Did you get $\frac{-2z+25}{3(z-3)}$?

Example 5 $\frac{3}{z^2-25} - \frac{1}{5+z} - \frac{z+1}{5-z}$

$$\frac{3}{(z+5)(z-5)} - \frac{1}{(z+5)} - \frac{(z+1)}{-1(z-5)}$$

$$\frac{3}{(z+5)(z-5)} - \frac{1}{(z+5)} \cdot \frac{(z-5)}{(z-5)} - \frac{(z+1)}{-1(z-5)} \cdot \frac{(z+5)}{(z+5)}$$

$$\frac{3}{(z+5)(z-5)} - \frac{1(z-5)}{(z+5)(z-5)} + \frac{(z+1)(z+5)}{(z-5)(z+5)}$$

$$\frac{3-1(z-5)+(z+1)(z+5)}{(z+5)(z-5)}$$

$$\frac{3-z+5+z^2+5z+z+5}{(z+5)(z-5)}$$

$$\frac{z^2+5z+13}{(z+5)(z-5)}$$

Practice 5 $\frac{z-23}{z^2-z-20} - \frac{2}{5-z}$

Did you get $\frac{3}{z+4}$?

Example 6 $\frac{5}{x-2} - \frac{3}{x+2}$

$$\frac{5}{(x-2)} \cdot \frac{(x+2)}{(x+2)} - \frac{3}{(x+2)} \cdot \frac{(x-2)}{(x-2)}$$

$$\frac{5(x+2) - 3(x-2)}{(x+2)(x-2)}$$

$$\frac{5x+10-3x+6}{(x+2)(x-2)}$$

$$\frac{2x+16}{(x+2)(x-2)}$$

$$\frac{2(x+8)}{(x+2)(x-2)}$$

Practice 6 $\frac{2}{x+3} - \frac{4}{x-3}$

Did you get $\frac{-2(x+9)}{(x+3)(x-3)}$?

Problems

1) $\frac{5x}{x+1} + \frac{3}{x+1} - \frac{2x}{x+1}$

2) $\frac{4y}{y+3} + \frac{4}{y+3} - \frac{y-5}{y+3}$

3) $\frac{x+2}{6x-42} - \frac{x-3}{5x-35}$

4) $\frac{3}{x+4} + \frac{3x-4}{x^2-16}$

5) $\frac{8}{x^2-9} + \frac{2}{x-3}$

6) $\frac{x+1}{x-1} - \frac{3x^2-1}{1-x^2}$

7) $\frac{x+5}{x-5} - \frac{x-5}{x+5}$

8) $\frac{a-2}{(a+2)^2} + \frac{a}{5(a+2)} - \frac{1}{25}$

Review: Meet with a tutor to verify your work on this worksheet and discuss some of the areas that were more challenging for you. If necessary, choose more problems from the homework to practice and discuss with the tutor.

For Tutor Use: Please check the appropriate statement:

_____ Student has completed worksheet but may need further assistance. Recommend a follow-up with the instructor.

_____ Student has mastered topic.